Counting and Cardinality Beginning With Subitizing

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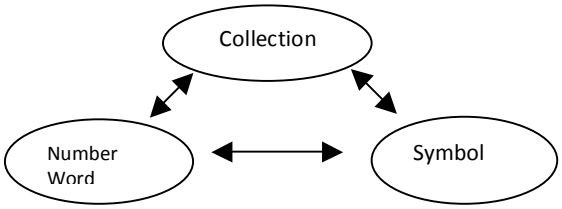
For years math educators have talked about the importance of students developing number sense. The National Council of Teachers of Mathematics defined five components of this understanding in the Curriculum and Evaluation Standards for School Mathematics: number meaning, number relationships, number magnitude, operations involving numbers and referents for numbers and quantities. (1989) Acquiring a sense of number establishes a general intuition about numbers and their relationships and is the foundation for learning mathematics with understanding. Children develop a sense of number through their experiences. According to Steffe, et al (1988) interpreting a number requires three pieces of information: a known collection; the word that represents the numerical value of the collection; and, the symbol used to record the number word. These three "bits" of information comprise six relationships as indicated by the arrows in the diagram below. Children must know these relationships for each number they learn.



When students see a number they must have a conceptual vision of a collection of objects that match the number and the number word that refers to that conceptual unit. This is the beginning of unitizing. Understanding these six relationships is neither a simple nor insignificant contribution to the mathematical success of students. The foundation for this complex and essential understanding begins in the early years of a child's mathematical development. The Common Core State Standards assign to kindergarten the responsibility for teaching counting and cardinality, the beginning of number sense. Therefore, teachers at this level must help students create "mind pictures" of the quantity of a number, its number word, and the corresponding collection.

If students are to work with arithmetic operations, having these images of numbers are critical. Von Glasersfeld describes the association of perception of composite patterns, such as the eight dots on a die, as the birth of the concept of number. The significance of the meaning of this is exemplified by the series of words and pictures. When we see the word "apple," we have no vision of what an apple looks like unless we have experienced it in reality. When we see the black and white drawing of the apple we have a little more information about what defines an apple. Seeing the colored picture of the apple gives us even more information. However, until we see, smell, touch, and taste a real apple, we do not understand what it is. We can liken this analogy to children's experiences with numbers. The abstract numeral provides extremely limited information. The number word is similarly abstract. A picture of a collection representing the numeral and the number word make the meaning of the number a little more real. However, working with a collection of objects provides experiences that make numbers and their value "real." These experiences give mental images of numbers and their meaning. This foundational knowledge enables children to understand, make sense of, and operate with numbers.

apple

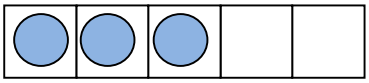
In the early grades students need to begin seeing numbers as "a group" (unitizing) if they are to understand and work successfully with numbers. In 1999 Doug Clements defined two types of subitizing, perceptual and conceptual, in his article, *Subitizing: What Is It? Why Teach it?* Clement describes ***perceptual subitizing*** as instantly recognizing the number of dots on the die without counting them individually. Children do not use other mathematical processes in this phase of subitizing. It results in students creating units (single things) to count. This means that students are keeping some of the dots separate from the others and counting them as a unit, such as recognizing two sets of three dots on a six die but counting them as a quantity of six.

On the other hand ***conceptual subitizing*** is the ability to break apart a group of dots into smaller groups and still not count the dots individually but recognize the total quantity of dots. When students see the same

two sets of four dots on an eight domino, they recognize them as a total set of eight. When children see dots in this way, they recognize numbers as units of units. This understanding assists students in developing abstract numbers and computational strategies. It is difficult for students who cannot subitize to conceptualize computational strategies (Steffe and Cobb,1988).

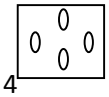
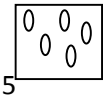
When students only "count" without a conceptual image of the number word, they engage in a meaningless exercise of calling numbers that are one more than the last. To prevent this from happening, teachers need to engage students in experiences that help children subitize. The word subitizing comes from the Italian word "subito" which means "immediately" or "right now." Initially students need experiences in recognizing small numbers, less than 5, from pattern arrangements of dots. Die or dominos with familiar dot patterns are useful in introducing students to perceptual subitizing, immediately recognizing numbers without counting. When this concept is firmly established, students need to recognize smaller numbers within larger ones (conceptual subitizing). An example is seeing a six die and sorting the dots into two sets of three dots that combine to make six.

A Five Frame is another tool for teaching subitizing. The teacher shows students a Five Frame with some of the cells filled with dots. The student describes the number of cells that are filled and the number that are unfilled to see the relationship between the two scenarios and a total quantity of five.



Dominoes are a third tool for helping students imprint small number quantities in their memory for recall in working with larger numbers and computing with efficiency. When using dominos, show students a single domino for a brief period of time and have them immediately tell the number of dots. You need to cover one end of the domino to avoid confusion about which set of dots are the focus. Later display both ends of the domino and have students practice number facts by using subitizing to count on to name the total number of dots on both ends of the domino and to recognize part-whole relationships among numbers. This step supports students in understanding that a number is made up of other numbers that is essential in composing and decomposing numbers fluently.

Having students make and use dot cards build an understanding of subitizing. Children need to see small numbers in random arrangements as well as in pattern arrangements. Examples are below:

Card 1  Card 2  Card 3  Card 4  Card 5 

Different spatial arrangements of the dots increase the difficulty for students. The most familiar patterns are Different spatial arrangements of the dots increase the difficulty for students as shown in the cards above. The most familiar patterns are simple to recognize. A slightly more difficult organization is in a linear pattern. Then a circular organization is more difficult than the linear pattern and the most challenging is a random arrangement (Beckwith and Restle, 1966, Wang, Resnick, and Boozer, 1971).Knowing these progressions of difficulty helps the teacher determine how to support students' learning to subitize. Teachers should assess students' level of recognition and plan subsequent activities to move them beyond their current level of understanding.

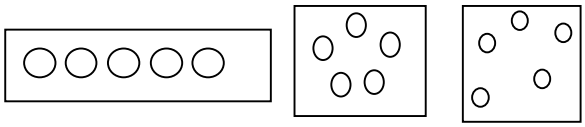
The dots cards may be used as a concentration game with a pair or small group of students. Cards are placed face-down on the table. Students turn over two cards. If the cards match in number and arrangement, the student takes the two cards. If the cards do not match in both number and arrangement, they are returned face-down and remain in play. The rules for determining a winner may be established by the teacher or the students. Engaging students in these types of activities ascertains that they connect numbers, number words, and collections in the six relationships described earlier in this article. They give mental images of the

quantitative value of numbers. This is an essential understanding if students are to engage in quantitative reasoning.

Monitoring students' understanding of subitizing is indispensable for the teacher to determine next instructional steps. Some possible guidelines follow:

- If the student recognizes the number word but not the number in the collection, the teacher can provide support by having students make sets of a given number, have students count sets of varying quantities, and to record the number in small collections. One-to-one counting is significant at this stage of development. Children should practice reading, writing, and matching sets with symbols using objects and dot cards. When students are able to subitize to five, confirm with an interview that the student is confident in the counting for numbers to five. Children operating at this stage should be able to work with a given number in multiple ways. They should work from the collection to the number word to the symbol and the reverse relationships as defined in the diagram on page 1.
- Play a game with the whole class, with small groups, or with individuals by hiding a given number of objects less than five. Show students the set of five objects. Cover part of the collection and ask students to name the number of objects that are hiding. This will build a sense of the relationship between the hidden and exposed objects as compared with the quantity of five. Once students are competent and confident with numbers to five begin work with numbers to 10. This can be done with Ten Frames to compare the number of dots as compared with a total quantity of ten. Students will also need to count on from numbers other than 1 and count backwards. These experiences build an understanding of part-whole relationships up to 10, combinations for ten, and composing and decomposing numbers within 10.
- Children need much practice with identifying multiple combinations for 10 such as 8 + 1 +1, 7 +2 +1, 4 + 3 +3, and so forth. This can be accomplished with using dots of different colors placed on a Ten Frame to show the various combinations for ten. When students are competent with combinations of 10, they will name the numbers without counting. A next step would be to use two Ten Frames to explore combinations to 20. Have students to name the numbers by saying 10 and 6, etc. to reinforce place value concepts. Other tasks might include working with a bead frame (pictured below) to show numbers to five, ten, and beyond. Remember that students still need to work with concrete materials, dot cards, Five Frames, Ten Frames, and eventually to open number lines.

To summarize, subitizing is fundamental to children's understanding of number. This means that students have mental images of pattern arrangements of numbers, linear arrangements, circular arrangements, and scattered (random) arrangements of objects. This contributes to children's understanding and use of the properties of number. This knowledge enables them to unitize, count on, compose, decomposes, conserve, compute, and make sense of place value.



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